

Black hole formation from collisions of cosmic fundamental strings

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ABSTRACT: We develop the general formalism for joining, splitting and interconnection of closed and open strings. As an application, we study examples of fundamental cosmic string collisions leading to gravitational collapse. We find that the interconnection of two strings of equal and opposite maximal angular momentum and arbitrarily large mass generically leads to the formation of black holes, provided their relative velocity is small enough.

KEYWORDS: String theory and cosmic strings, Black Holes in String Theory, Long strings.

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1. Introduction

Recent works have indicated that in brane inflation models cosmic strings are copiously produced during the brane collision [1, 2]. This has led to a renewed interest in the physics of cosmic strings and to consider the exciting possibility that there could be long-lived fundamental strings of cosmic size (for reviews, see [3–6] and references therein). Finding such objects could constitute a test of string theory.

The dynamics of cosmic strings could lead to interesting astrophysical events such as gravitational waves or black hole formation. Some aspects of the dynamics of cosmic string interactions were studied in [7, 8] (for Abrikosov-Nielsen-Olesen strings, see recently [9]).

Here we will develop in full detail the classical formalism of string splitting, joining and intercommutation. Our formulas (appendix) provide explicit expressions for the outgoing string solution starting with an arbitrary initial string configuration before interaction. Understanding the dynamics and the different features of splitting and joining processes is of interest, since these processes are the basis of the interaction rules in string theory.

As an application, we will study the process of possible gravitational collapse as a result of the collision of cosmic fundamental strings. Surprisingly, we will find that gravitational collapse is a quite common phenomenon ensuing the encounter of strings of equal and

opposite maximal angular momentum, which classically are rotating straight strings, and folded in the case of the closed string. If the initial strings just touch at the end points, then they can join forming one single string. If they meet at some intermediate point, then they can interconnect giving rise to two new strings.

We then study the evolution of the resulting strings by the standard flat spacetime dynamics. We find that they typically contract in a finite time to a minimum size, which sometimes is smaller than the gravitational radius R_s .

If the strings meet with zero relative transverse momentum, we find that, for generic values of the intersecting positions and angle, a finite fraction of the mass of the resulting interconnected strings collapses into a mathematical point.

In any of these situations, gravitational forces become very strong when the string size approach R_s and should enhance the evolution towards the collapse, ensuing in the formation of a horizon and hence a black hole [10–12] (other discussions can be found e.g. in [13, 14]). In our computation the mass (proportional to the length) of the strings appear as an overall scale, therefore this phenomenon can occur for arbitrarily large values of the mass.

If the transverse momentum is not zero, then the resulting strings will be stretched in the transverse direction. The size is of the order of the length times the relative transverse velocity v . For non-relativistic relative motions with v much less than the product of the gravitational constant times the string tension, this size will be much smaller than the gravitational radius. We conclude that also in this case, the interconnection of our strings generically leads to the formation of a black hole.

We also consider a long-lived version of the string with maximum angular momentum. This is a closed string with some component in extra dimensions, whose motion in $3 + 1$ dimensions is the same as the open (or folded) string with maximum angular momentum. We discuss for which range of the parameters and for which magnitudes of cross sections black hole formation is to be expected.

2. New examples of long-lived cosmic strings

We consider type II strings in the presence of D branes or with extra dimensions compactified on a torus. We are interested in constructing cosmological strings where breaking processes are suppressed, leading to long-lived configurations. Such strings decay primarily by emission of soft gravitational radiation.¹

2.1 Rotating straight string on $M^4 \times S^1$

Let t, X, Y, Z represent uncompact coordinates of M^4 (3+1 dimensional Minkowski space)

¹The most stable massive non-BPS closed string in type II string theories seems to be a circular string rotating in two or more planes [15], for which breaking is maximally suppressed and radiation is feeble. We will not study this string in this paper.

and W compact dimensions of radius R . The solution is as follows:

$$\begin{aligned}
 X &= L \cos \tau \cos \sigma, & X_R(\sigma_-) &= \frac{1}{2} L \cos \sigma_-, & X_L(\sigma_+) &= \frac{1}{2} L \cos \sigma_+, \\
 Y &= L \sin \tau \cos \sigma, & Y_R(\sigma_-) &= -\frac{1}{2} L \sin \sigma_-, & Y_L(\sigma_+) &= \frac{1}{2} L \sin \sigma_+, \\
 W &= nR\sigma, & W_R(\sigma_-) &= \frac{1}{2} nR\sigma_-, & W_L(\sigma_+) &= \frac{1}{2} nR\sigma_+, \\
 t &= \kappa \tau, & \kappa &= \sqrt{L^2 + n^2 R^2},
 \end{aligned} \tag{2.1}$$

where $\sigma_{\pm} = \sigma \pm \tau$, $\sigma \in [0, 2\pi)$ and n is an integer representing the winding number.

The solution is classically unbreakable for $n = 1$. This can be seen as follows. The closed string can break only if at some given time (say $\tau = 0$) there are two different points of the string which get in contact, i.e. $\vec{X}(\sigma_1) = \vec{X}(\sigma_2)$ for the non-compact coordinates, and for the compact coordinate $W(\sigma_1) = W(\sigma_2) + 2\pi k R$ for some k . Using this condition for X and Y we find $\sigma_1 = 2\pi - \sigma_2$. Now the condition for W gives $\pi - \sigma_2 = k\pi/n$. For $n = \pm 1$ this gives $\sigma_1 = \sigma_2 = 0 \pmod{2\pi}$, which means that the string cannot break because the two points coincide and thus there is no string left between them. For $|n| > 1$, one always has at least one solution with $\sigma_1 \neq \sigma_2$, and the string can break.

A well known stable string on $M^4 \times S^1$ is the BPS string [16, 17] $\vec{X} = \vec{X}_L(\sigma_+)$, $W = nR\sigma + 2\alpha' m/R$. In our case, the mechanism for stability is different. Although the string looks folded in 3+1 dimensions – and in fact it looks identical to the unstable rotating string of maximal angular momentum [18, 19] – it cannot break classically because all points of the string are separated in the internal dimension W . If $R \gg \sqrt{\alpha'}$, then breaking by quantum effects is also suppressed. As mentioned above, it can decay by radiation, with a rate (in four dimensions) [19] $\Gamma \sim g_s^2 M$, $M \sim \mu L$, where $\mu = (2\pi\alpha')^{-1}$ is the string tension and g_s is the closed string coupling constant. The radiation is dominated by soft modes with emitted energy $\omega \sim 1/L$. Thus

$$-\frac{dM}{dt} \sim \Gamma \times \omega \cong c_0 g_s^2 \mu, \tag{2.2}$$

where c_0 is a numerical constant of order one. Therefore the string takes a time $\sim M/g_s^2$ (or $\sim L/g_s^2$) to substantially decrease its mass.

2.2 Rotating open string which oscillates in extra dimensions

Consider a brane-world model, with a D3-brane placed in the three uncompact directions X, Y, Z of our universe. Let W stand for an extra dimension. The open string solution with Dirichlet boundary conditions at $W = 0$ is given by

$$\begin{aligned}
 X &= L \cos \theta \cos \tau \cos \sigma, & Y &= L \sin \tau \cos \sigma, \\
 W &= L \sin \theta \sin \tau \sin \sigma, & t &= L \tau, & L &= 2\alpha' M.
 \end{aligned} \tag{2.3}$$

This solution is the analog of the squashing closed string of [19] in the case of an open string. In fact, it has the same form, but here $\sigma \in [0, \pi)$ whereas for the closed string $\sigma \in [0, 2\pi)$.

The solution represents a string rotating in the plane X, Y , with the ends attached on the brane $W = 0$, which at the same time oscillates in the extra dimension W (with maximum amplitude $\pm L \sin \theta$ for the middle point). The string can break only at the special times where it lies on the brane $W = 0$, namely $\tau = n\pi$, as otherwise it is impossible to get Neumann conditions on the free endpoints in the W direction. The time between two successive events, when much of string lies within a width $\sim l_s \equiv \sqrt{\alpha'}$ from the brane, is of the order of L . Therefore, for L large and assuming $L \sin(\theta) \gg l_s$, this string lives a long time $\sim L \sim M$, before breaking into massive pieces. In the following, we will consider the interactions of strings during this time interval, in which the breaking is exponentially suppressed, since it is classically forbidden.

As an aside remark, we can note that during the time $\sim l_s$ in which the string lies on the brane, it will break with a probability that using the rules of [19] is found to be $\sim g_o^2 L / l_s$, where g_o is the open string coupling constant and $g_o^2 = g_s$. If this is much less than one, it may take several cycles (a number of order $l_s / (g_o^2 L)$) for the string to break, each cycle lasting a time $\sim L$.

The string nevertheless loses energy at all times by gravitational radiation.² The decay rate by massless emission can be estimated following the analysis of [15, 19]. In these works the classical radiation rate was found to be a function of L and of the emitted energy ω of the following factorized form:

$$Rate(N_0) \sim L^{5-D} \cdot F(N_0) . \tag{2.4}$$

where D is the number of uncompact spacetime dimensions and $F(N_0)$ is a decreasing function of the integer $N_0 \equiv L\omega$. Therefore, for large L , the massless emission is concentrated at small ω , where the classical result is expected to hold. The highest decay rate occurs in the case of $D = 4$, since spaces with $D > 4$ have a decay rate suppressed by inverse powers of L . For $D = 4$, by summing over N_0 we find that

$$\Gamma_{\text{graviton}} \sim g_s^2 \sqrt{N} , \quad N = \alpha' M^2 . \tag{2.5}$$

Since, as in the previous example of section 2.1, the radiation is dominated by soft modes with emitted energy $\omega \sim 1/L$, the lifetime required for a substantial decrease of the energy is again of order M/g_s^2 . In spaces with $D > 4$, this string lives an even longer time.

3. Black hole formation

3.1 By joining of strings

In this subsection we show an example of a process where two long strings join by their ends and the resulting string becomes very small (in fact, pointlike) during the evolution.

First, consider two open strings with maximum angular momentum, zero linear momentum and equal energies described by the solutions: $X_{I,II}(\tau, \sigma) = X_{I,III}(\tau + \sigma) +$

²Emission of vector bosons can be shown to be suppressed with respect to graviton emission by inverse powers of L .

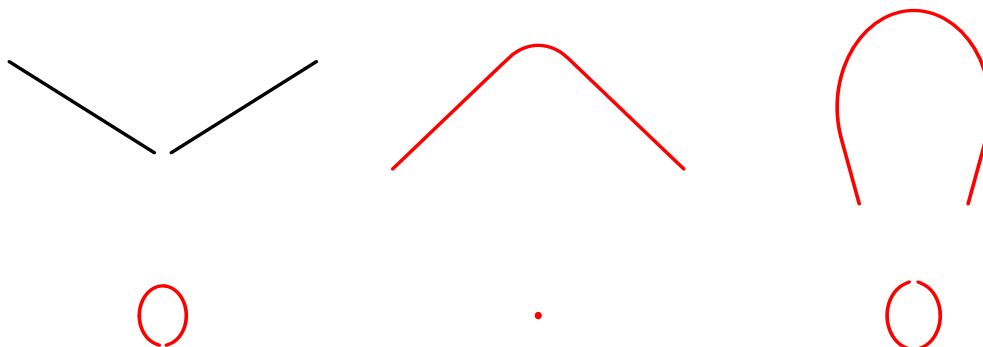


Figure 1: Evolution of the open string which results from the joining of two open strings with maximum and opposite angular momentum.

$X_{\text{I,II}}(\tau - \sigma)$ with

$$\begin{aligned}
 X_{\text{I}}(\sigma, \tau) &= L \cos \sigma \cos \tau, & Y_{\text{I}}(\sigma, \tau) &= L \cos \sigma \sin \tau, \\
 X_{\text{II}}(\sigma, \tau) &= 2L + L \cos \sigma \cos \tau, & Y_{\text{II}}(\sigma, \tau) &= -L \cos \sigma \sin \tau, \\
 X_{\text{IL}}(s) &= \frac{L}{2} \cos s, & Y_{\text{IL}}(s) &= \frac{L}{2} \sin s \\
 X_{\text{IIL}}(s) &= L + \frac{L}{2} \cos s, & Y_{\text{IIL}}(s) &= -\frac{L}{2} \sin s
 \end{aligned} \tag{3.1}$$

The strings I and II have equal and opposite angular momenta given by $J_{\text{I}} = L^2/\alpha'$, $J_{\text{II}} = -L^2/\alpha'$. As they rotate, the end $\sigma = 0$ of the string I touches the end $\sigma = \pi$ of the string II at $\tau = n\pi$, $n = \text{integer}$.

Consider the situation where the strings join at $\tau = 0$. The resulting open string solution has $J = 0$, since angular momentum is conserved and the original total angular momentum of the system is zero. By applying the formulas of appendix A, we find the solution after the joining: $X(\tau, \sigma) = X_L(\tau + \sigma) + X_L(\tau - \sigma)$ with

$$\begin{aligned}
 X_L(s) &= \begin{cases} L + \frac{L}{2} \cos 2s & -\frac{\pi}{2} \leq s < \frac{\pi}{2} \\ -\frac{L}{2} \cos 2s & \frac{\pi}{2} \leq s < \frac{3\pi}{2} \end{cases} \\
 Y_L(s) &= -\frac{L}{2} \sin 2s.
 \end{aligned} \tag{3.2}$$

This solution is shown in figure 1. It describes an open string which at $\tau = 0$ is completely straight, then it bends and contracts until it becomes a point at $\tau = \pi/2$. The solution is periodic with period π .

Note that the joining process can occur at the lowest order in string perturbation theory (with probability $O(g_o^2)$) once one has got this initial configuration).

So far gravitational effects have not been taken into account. We start with two long straight strings $J_{\text{max}} + \text{anti}J_{\text{max}}$ which join forming an open string with $J = 0$ in a regime where its size is much larger than the gravitational radius R_s . In this regime, the evolution of the string is governed by the classical string equations of motion in flat spacetime. As

the open string reduces its size, gravitational effects become more and more important. A string which reduces to a point should clearly undergo gravitational collapse. This should happen when the size of the string becomes smaller than R_s . For a string of length $\ell \sim M/\mu$, where $\mu = 1/(2\pi\alpha')$ is the string tension, the gravitational radius is $R_s \sim 2G\mu\ell$. Therefore, when the string contracts by a factor of order $(G\mu)^{-1}$, gravitational collapse should be inevitable and a horizon will form [10–12].

One important question is whether the open string could radiate out most of its energy before its size becomes smaller than the Schwarzschild radius. There are two decay channels: the radiation channel, where the string emits a graviton, and the massive channel, where the string breaks into two pieces.

Let us first consider the massive channel. The evolution of the two pieces after the breaking can be followed by using the general formulas of appendix A. Because of momentum conservation, in the present case each piece will carry a momentum in the inward direction. Therefore the system cannot lose energy by breaking. This argument ignores gravitational effects. Taking into account the attractive nature of gravitational forces reinforces the fact that each piece will follow an inward collapse.

The radiated energy can be estimated with the rules of [19]. For a smooth string in four dimensions (and even if there are kinks), the mass loss rate is given by eq. (2.2). For this string, $t = 2\alpha' M\tau = \frac{M}{\pi\mu}\tau$. As the string loses mass, the value of M is changing. This process is very slow for large M , so we can follow it adiabatically and assume that at each time the string is described by the same solution with $M(t)$. We then write $dt = \frac{1}{\pi\mu}(Md\tau + \tau dM)$. Hence

$$-\frac{dM}{M} \cong \frac{c_0 g_s^2 d\tau}{\pi + c_0 g_s^2 \tau} . \tag{3.3}$$

The total energy radiated from the initial configuration until the string becomes a point is obtained by integrating this equation from $\tau = 0$ to $\tau = \pi/2$. We get

$$M\left(\tau = \frac{\pi}{2}\right) = \beta M(\tau = 0), \quad \beta = \left(1 + \frac{c_0 g_s^2}{2}\right)^{-1} . \tag{3.4}$$

Since c_0 is a number of order 1, this mass is of the same order of the initial mass. This shows that a black hole will be formed before the string becomes a point.

The starting point of the above example involves two open strings, which we know to be unstable. The same process can occur for the long-lived strings of section 2.

In the case of the joining of two rotating open strings which oscillate in extra dimensions (section 2.2), when the amplitude of oscillation is much smaller than the length of the string (corresponding to the parameter $\theta \ll 1$ in eq. (2.3)), the dynamics of the joining process is essentially the same as in the above example (at the same time, the amplitude of oscillation must be much larger than $\sqrt{\alpha'}$ to suppress breaking by quantum effects).

In the other case, that is considering the joining of two rotating straight closed strings on $M^4 \times S^1$ with winding number $n = 1$ (section 2.1), one can easily see that the joining equations for the X, Y coordinates are the same as for the open string case seen above. It

is also easy to see that the joining conditions are satisfied in the W coordinate, giving rise to two possible solutions, with winding 0 or with winding equal to 2.

As an aside remark, we note that, as far as the flat space evolution is concerned, as the string (3.2) is contracting, the ends of the string approach each other, and they touch only in the limit that the string is a point. Although in this limit quantum and gravitational effects are important, it is nevertheless interesting to follow the classical world-sheet evolution. When the ends of the string touch, there is a certain probability given by the coupling constant g_o^2 that they join forming a closed string. The resulting solution is obtained by defining $\vec{X}_L(s)$ and $\vec{X}_R(s)$ to be equal to the open string $\vec{X}_{L,R}$ at $\tau = \pi/2$ and, for $\tau > \pi/2$, one imposes closed string boundary conditions, $\vec{X}(\tau, \sigma + 2\pi) = \vec{X}(\tau, \sigma)$. Remarkably, the resulting solution is the pulsating circular string solution described by (we set the center of mass coordinate to zero)

$$\begin{aligned} X(\sigma, \tau) &= 2L \cos \sigma \cos \tau, & Y(\sigma, \tau) &= 2L \sin \sigma \cos \tau, \\ X_L(s) &= X_R(s) = L \cos s, & Y_L(s) &= Y_R(s) = L \sin s. \end{aligned} \tag{3.5}$$

Another remark is that the circular pulsating string can be obtained from the quantum scattering of two gravitons. In ref. [19] it was remarked that the quantum amplitude for the process *two gravitons* \leftrightarrow *pulsating string* is the same as the amplitude for *two gravitons* \leftrightarrow *Jmax string*. The rate of the last process was computed in ref. [20] and checked in ref. [21]. From these results, we find the rate for the present process of two gravitons forming a pulsating circular string: $\Gamma \sim g_s^2 e^{-\frac{1}{2}\alpha' M^2(\log(4)-1)}$ where M is the mass of the pulsating string. Once the circular pulsating string is formed, it should inevitably collapse into a black hole (if it is not already inside the horizon, it will shrink with a negligible probability of breaking [19]). Therefore, this process provides an example of a first-principle calculation based on string perturbation theory of black hole formation. The cross-section for that particular final state is exponentially small. In four spacetime dimensions: $\sigma \sim \alpha' g_s^2 e^{-\frac{1}{2}\alpha' M^2(\log(4)-1)}$.

3.2 By interconnection of two strings

In the previous example, black hole formation requires a special initial configuration such that the endpoints of the two strings touch during the evolution. A more generic process is the case of string interconnection.³

When two fundamental strings cross, there is a probability given by the string coupling that the strings will interconnect, as in figure 2. As shown in the figure, there are two possible ways that the string can interconnect. This is a common process in 3+1 dimensions, where two infinitely long strings always cross for generic initial data. For finite-size strings, the collision has a cross section of the order of the square of the length of the string.

An interesting question is what is the probability that a black hole is formed as a result of the collision. Computing this from string perturbation theory is obviously very complicated, so we will try to address this question by means of the following experiment:

³For open strings, this process corresponds to the u-channel open string diagram (we thank D. Amati for a discussion on this point).

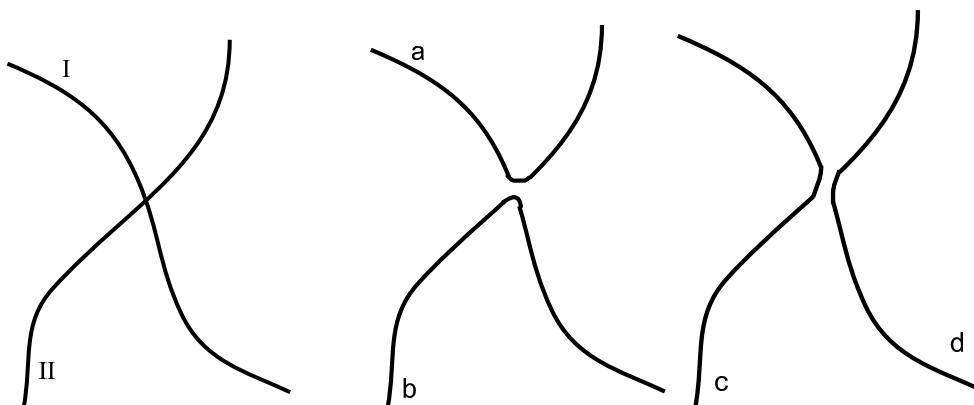


Figure 2: Interconnection process. When two strings cross, there are two possible ways that they can interconnect, leading to strings *a* and *b* or strings *c* and *d*.

we send two straight rotating strings against each other, with random position for the center of mass coordinates and random value for the relative orientation (within the range where the interconnection is possible). After repeating the experiment N_e times, we ask how many of the resulting string configurations are black holes. We will consider several conditions for black hole formation. One condition is that one of the two final strings completely lie inside its Schwarzschild radius R_s at some time during the evolution (ignoring detailed features due to the angular momentum). Another condition is that at some time the average size of the string lies inside its Schwarzschild radius. Finally, a third condition, is that a segment of the string lies within the Schwarzschild radius. In our study, the reduction to a small size just follows by the natural shrinking of the string that results from flat space evolution, without taking into account gravity. In any of these three situations, gravitational forces become very strong when the string size approach R_s and should enhance the evolution towards the collapse.

Consider first the interconnection of two rotating open strings of the type described in section 2.2. They rotate in the plane X, Y , oscillate in the extra W dimension, and they may also have transverse momentum in the Z direction on the brane. We will consider the case of opposite angular momentum in the X, Y plane. After interconnection, the two emerging strings also spread in the Z direction. When the center-of-mass transverse Z -motion of the string is non-relativistic, the spread in the Z direction can be neglected as compared to the spread in the X, Y direction. In addition, as discussed in the previous joining case, when the amplitude of oscillation in the W direction is much smaller than the length of the string (corresponding to the parameter $\theta \ll 1$), the dynamics of the interconnection process is essentially the same as that of the interconnection of two open strings of maximum angular momentum. Therefore, to simplify the discussion, we will first consider the case of two open strings with equal and opposite maximum angular momenta lying at $Z = 0$ and $W = 0$.

3.2.1 The solutions after the interconnection

Consider two open strings of (opposite) maximal angular momentum in the XY plane,

having the same energy, which cross at some angle at $\tau = 0$. We will take the gauge $t = L\tau$. The solutions are, with $0 \leq s_{\pm} = \frac{\sigma \pm \tau}{L} \leq 2\pi$,

$$\begin{aligned} X_{IL} &= \frac{L}{2} \cos(s_+), & X_{IR} &= \frac{L}{2} \cos(s_-) \rightarrow X_I = L \cos\left(\frac{\tau}{L}\right) \cos\left(\frac{\sigma}{L}\right) \\ Y_{IL} &= \frac{L}{2} \sin(s_+), & Y_{IR} &= -\frac{L}{2} \sin(s_-) \rightarrow Y_I = L \sin\left(\frac{\tau}{L}\right) \cos\left(\frac{\sigma}{L}\right) \end{aligned} \quad (3.6)$$

$$\begin{aligned} X_{IIL} &= \frac{A}{2} + \frac{L}{2} \cos(s_+ + \alpha), & X_{IIR} &= \frac{A}{2} + \frac{L}{2} \cos(s_- - \alpha) \\ &\rightarrow X_{II} = A + L \cos\left(\frac{\tau}{L} + \alpha\right) \cos\left(\frac{\sigma}{L}\right) \\ Y_{IIL} &= \frac{B}{2} - \frac{L}{2} \sin(s_+ + \alpha), & Y_{IIR} &= \frac{B}{2} + \frac{L}{2} \sin(s_- - \alpha) \\ &\rightarrow Y_{II} = B - L \sin\left(\frac{\tau}{L} + \alpha\right) \cos\left(\frac{\sigma}{L}\right) \end{aligned} \quad (3.7)$$

A, B and α are constants parametrizing the center of mass coordinate of the string II and its relative orientation. We take $A > 0$.

The open strings are parametrized by $0 \leq \sigma \leq \pi L$. Their energy is

$$E = \frac{1}{2\pi\alpha'} \int_0^{\pi L} d\sigma \partial_\tau X^0 = L/\alpha'.$$

We assume that the two strings interconnect at $\tau = 0$. They intersect at σ_0 and σ'_0 respectively. The two strings I, II of equal length recombine forming two strings a, b (or c, d) of different lengths forming some kink.

The intersection equations are:

$$\begin{aligned} 0 &= B - L \sin(\alpha) \cos(\sigma'_0) \rightarrow L \cos(\sigma'_0) = \frac{B}{\sin(\alpha)} \\ L \cos(\sigma_0) &= A + L \cos(\alpha) \cos(\sigma'_0) \rightarrow L \cos(\sigma_0) = A + B \frac{\cos(\alpha)}{\sin(\alpha)} \end{aligned} \quad (3.8)$$

A necessary condition for the strings to intersect at $\tau = 0$ is $(A - L)^2 + B^2 \leq L^2$. The intersection equations $\vec{X}_{IIL}(\sigma'_0) + \vec{X}_{IIR}(\sigma'_0) = \vec{X}_{IL}(\sigma_0) + \vec{X}_{IR}(\sigma_0)$ imply

$$\vec{X}_{IIL}(\sigma'_0) - \vec{X}_{IL}(\sigma_0) = \vec{X}_{IR}(\sigma_0) - \vec{X}_{IIR}(\sigma'_0) \equiv \vec{Q} \quad (3.9)$$

Now we consider one of the two cases of interconnection shown in figure 2.

The two open strings $\vec{X}_{a,b}(\sigma, \tau)$ after interconnection will be described by a world-sheet parameter σ with interval of size $\Delta_a \sigma = \sigma_0 + \pi - \sigma'_0$ and $\Delta_b \sigma = \sigma'_0 + \pi - \sigma_0$ respectively (the periodicity interval for the Left and Right part being the double of the above). Their energy is $L(\pi + \sigma_0 - \sigma'_0)/2\pi\alpha'$ and $L(\pi + \sigma'_0 - \sigma_0)/2\pi\alpha'$.

We will find that $\vec{X}_{a,b}(\sigma, \tau)$ have momentum and, as in appendix A, we will write

$$\vec{X}_{a,b;L,R}(s) = [\vec{X}_{a,b;L,R}(s) \mp \vec{k}_{a,b}s]_{(0,2\Delta_{a,b}\sigma)} \pm \vec{k}_{a,b}s \quad (+ \text{ for } L \text{ and } - \text{ for } R) \quad (3.10)$$

where we define the periodic function $[f(s + 2\Delta\sigma)]_{(0,2\Delta\sigma)} = [f(s)]_{(0,2\Delta\sigma)}$. In physical units the momenta are $\vec{p}_{a,b} = \vec{k}_{a,b}\Delta_{a,b}\sigma/\pi\alpha'$.

The string a is (period $2\Delta_a\sigma = 2\pi - 2\sigma'_0 + 2\sigma_0$)

$$\vec{X}_{aL,R}(s) = \begin{cases} \vec{X}_{IL,R}(s), & -\sigma_0 \leq s \leq \sigma_0 \\ \vec{X}_{IIL,R}(s - \sigma_0 + \sigma'_0) \mp \vec{Q}, & \sigma_0 \leq s \leq 2\pi - 2\sigma'_0 + \sigma_0 \end{cases} \quad (3.11)$$

Further:

$$\vec{k}_a = \frac{1}{2\Delta_a\sigma}(\vec{X}_{IIL}(2\pi - \sigma'_0) - \vec{Q} - \vec{X}_{IL}(-\sigma_0)) = -\frac{1}{2\Delta_a\sigma}(\vec{X}_{IIR}(2\pi - \sigma'_0) + \vec{Q} - \vec{X}_{IR}(-\sigma_0))$$

Explicitly

$$k_a^x = \frac{L}{2\Delta_a\sigma} \sin(\alpha) \sin(\sigma'_0), \quad k_a^y = \frac{L}{2\Delta_a\sigma} (\sin(\sigma_0) + \cos(\alpha) \sin(\sigma'_0)),$$

such that $[\vec{X}_{aL,R}(s) \mp \vec{k}_a(s + \sigma_0)]$ take the same value at $s = -\sigma_0$ and $s = 2\pi - 2\sigma'_0 + \sigma_0$.

The open string $\vec{X}_a(\sigma, \tau) = \vec{X}_{aL}(\sigma + \tau) + \vec{X}_{aR}(\sigma - \tau)$ is defined for $0 \leq \sigma \leq \Delta_a\sigma$. One can check that $\partial_\sigma \vec{X}_a = 0$ for $\sigma = 0, \Delta_a\sigma$ and any τ .

The string \vec{X}_b is obtained by interchanging $X_{IL,R} \leftrightarrow X_{IIL,R}$ and $\sigma_0 \leftrightarrow \sigma'_0$ in the formulas for \vec{X}_a . We get $\Delta_b\sigma = \pi + \sigma'_0 - \sigma_0$,

$$k_b^x = -\frac{L}{2\Delta_b\sigma} \sin(\alpha) \sin(\sigma'_0), \quad k_b^y = -\frac{L}{2\Delta_b\sigma} (\sin(\sigma_0) + \cos(\alpha) \sin(\sigma'_0)).$$

As expected, the momenta are equal and opposite, that is $\vec{p}_a = \vec{k}_a\Delta_a\sigma/\pi\alpha' = -\vec{p}_b = -\vec{k}_b\Delta_b\sigma/\pi\alpha'$ and the energy is conserved

$$E = 2\frac{1}{2\pi\alpha'} \int_0^\pi d\sigma \partial_\tau X^0 = \frac{1}{2\pi\alpha'} \int_0^{\Delta_a\sigma} d\sigma \partial_\tau X^0 + \frac{1}{2\pi\alpha'} \int_0^{\Delta_b\sigma} d\sigma \partial_\tau X^0.$$

Note that the interconnecting equations contain as a particular case the joining considered in the previous section, which is formally obtained for $\sigma_0 = 0$ and $\sigma'_0 = \pi$.

The other possible pair shown in figure 2, $\vec{X}_{c,d}(\sigma, \tau)$, can be constructed in a similar way. The strings will have energy $E_{c,d} = \frac{\Delta_{c,d}\sigma}{2\pi\alpha'} L$, with $\Delta_c\sigma = \sigma_0 + \sigma'_0$, $\Delta_d\sigma = \pi - \sigma_0 - \sigma'_0$.

3.2.2 Black hole events

Having the solutions of the two outgoing strings after the interconnection, we now consider their evolution and study the possible black hole formation.

We first explore the possibility that the whole mass of the outgoing string collapses to a size less than the Schwarzschild radius. Specifically, in this subsection we examine two conditions for black hole formation:

1) At some time during the evolution the average size of the string,

$$\bar{R}^2 \equiv \frac{1}{\Delta\sigma_{a,b}} \int_0^{\Delta\sigma_{a,b}} d\sigma R^2(\sigma), \quad (3.12)$$

N_e	$G\mu$	$N_{bh} (\bar{R} < R_s)$	$N_{bh} (R(\sigma) < R_s)$
10000	10^{-2}	1900 – 2000	1100 – 1200
10000	10^{-3}	300 – 320	95 – 110
10000	10^{-4}	40 – 46	1 – 3
50000	10^{-5}	20 – 30	0 – 4
50000	10^{-6}	3 – 5	0

Table 1: Number of black hole events in N_e string collisions.

$$R^2(\sigma) = (X_{a,b}(\sigma, \tau) - X_{a,b}^{\text{CM}})^2 + (Y_{a,b}(\sigma, \tau) - Y_{a,b}^{\text{CM}})^2,$$

is less than the Schwarzschild radius $R_s = 2GM$, where M is the mass of one of the outgoing strings a or b .

2) At some time during the evolution all points of the string lie within the Schwarzschild radius, i.e. $R(\sigma) < R_s$ for all σ .

The masses of the strings a and b are given by

$$M_{a,b} = L \frac{\Delta_{a,b}\sigma}{2\pi\alpha'} \sqrt{(1 - 4\vec{k}_{a,b}^2)}. \quad (3.13)$$

It is convenient to express R_s as

$$R_s = 2(G\mu) \frac{M}{\mu}, \quad \mu = \frac{1}{2\pi\alpha'}. \quad (3.14)$$

The fundamental string has a tension μ whose value could be anywhere between the TeV scale and the Planck scale. In brane inflation models, one expects a narrower range $10^{-12} < G\mu < 10^{-6}$.

We have followed the evolution of the strings after interconnection in N_e events taking random values for the center of mass coordinates A, B and for the relative orientation α (within the intersection range). We have seen that, when the strings shrink to a minimum size which is much smaller than the initial size, they typically have a shape describing an incomplete circle.⁴

The number of black hole events N_{bh} depend on the value of $G\mu$. Table 1 summarizes our results. We see that the condition $R(\sigma) < R_s$ for all σ gives less black hole events. This is due to cases where a small tail of the string lies outside the Schwarzschild radius.

We also observe that the distribution of black hole events in the region of possible parameters for the center of mass coordinates A and B is nearly homogeneous. From the data of table 1 one sees that N_{bh} (computed with either criterium) has a power-like dependence with $G\mu$.

A typical black hole event is shown in figure 3. The string after the interconnection has a kink at $\tau = 0$, which then separates into two kinks moving in opposite directions. If the two pieces that form the string have a comparable size, then the strings after the

⁴In the case of a mass distributed on a circle, a ‘‘Laplace’’ Schwarzschild radius R_s can be defined by requiring that the gravitational potential at the center is equal to $c^2/2$ (c speed of light). This gives $R_s = 2GM$, as in the case of the sphere studied by Laplace.

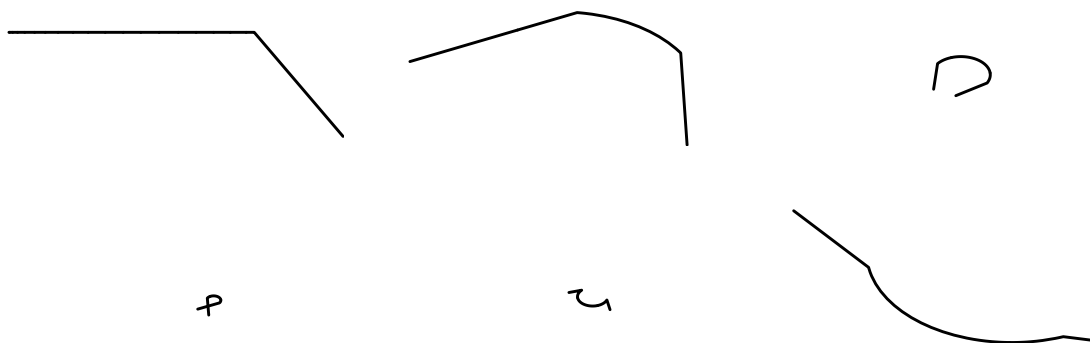


Figure 3: A string solution resulting from interconnection possibly leading to gravitational collapse, after shrinking by its own classical evolution. A generic feature is the formation of two kinks moving in opposite directions along the string. The figures are not schematic, they are obtained using the exact classical evolution of the string.

interconnection will have a small angular momentum. This is typically the situation leading to a contraction of the strings to very small size.

This figure, however, does not give information on how the mass is distributed. In fact, as we will see in the next section, at some $\tau = \tau_0$ an important fraction of the mass is always concentrated in one point. This fact, which is not possible to see in figure 3 (but can be seen in figure 4), implies that all cases of this collision of $J_{\text{max}} + \text{anti}J_{\text{max}}$ strings should lead to black hole formation. In particular, this indicates that the cross-section for the scattering of two long strings to form a black hole is essentially given by the geometric area of the overlap of the two strings, times $g_o^4 = g_s^2$, where g_s being the closed string coupling constant.

3.2.3 Inevitable collapse in a generic $J_I = -J_{II}$ case

In section 3.1 we have already seen that a black hole will form in the case of the *joinings* of J_{max} and $\text{anti}J_{\text{max}}$ strings. In that case, the string that results from the joining process shrinks, becoming a point at $\tau = \pi/2$. That is, just by the evolution dynamics in flat space, all the mass concentrates in a region of zero size. We argued that energy loss by radiation or breaking is negligible and inclusion of gravitational effects will reinforce the shrinking and finally a black hole will form.

In the case of interconnection, we first observe that the same phenomenon of the complete shrinking of the whole mass occurs when the interconnection takes place at $\sigma_0 = \sigma'_0 = \pi/2$ and $\alpha = 0$ (with zero momentum in the transverse direction), that is, when the interconnection takes place in the middle point at zero angle. In this case, the interconnected strings have momentum along Y . Again, the string after interconnection shrinks to zero size at $\tau_0 = \pi/2$, and the same previous argument applies with the conclusion that a black hole will form.

The underlying mechanism is the cancellation of the dependence in σ of the Left part with the Right part: in more detail, for that value of $\tau = \tau_0$, the Left piece that, by construction, equals the Left piece of X_{II} , cancels with the Right piece that equals X_I and *viceversa*.

Let us now consider the slightly more general case in which $\sigma_0 = \sigma'_0 = \pi/2$, but $\alpha \neq 0$. The periodicity interval in σ of the interconnected strings is again $\Delta\sigma = 2\pi$. We find that at $\tau_0 = \pi/2 - \alpha/2$ the dependence in σ cancels in the intervals $-\pi + \alpha/2 \leq \sigma \leq -\alpha/2$ and $\alpha/2 \leq \sigma \leq \pi - \alpha/2$. Since the energy, and the mass, of the string is uniformly distributed in σ , we see that for small α almost all of the mass shrinks to zero size. Also in this case a black hole will form with a mass of the same order of the mass of the incoming strings. In fact, the dimension of the incoming strings sets the overall scale, and thus the result holds for an arbitrary value of the mass.⁵

By the same reasons, even when α is not small, a finite fraction of the (arbitrarily large) mass will shrink to zero size and form a black hole.

One can further investigate the general case of generic values of $\sigma_0, \sigma'_0, \alpha$ (recall that the periodicity interval in σ of the string a is $2\Delta_a = 2\pi\sigma - 2\sigma'_0 + 2\sigma_0$ and of the string b it is $2\Delta_b\sigma = 2\pi - 2\sigma_0 + 2\sigma'_0$). The resulting strings have in general linear momentum in the XY plane and angular momentum as well. Taking for instance the string b , we find that a finite fraction of its arbitrarily large mass shrinks to zero size at

$$\tau_0 = \begin{cases} \frac{\pi}{2} - \frac{\alpha}{2} + \frac{1}{2}(\sigma'_0 - \sigma_0) & \text{if } \Delta_b\sigma > \alpha \\ \frac{\pi}{2} - \frac{\alpha}{2} - \frac{1}{2}(\sigma'_0 - \sigma_0) & \text{if } \Delta_b\sigma < \alpha \end{cases}$$

This fraction is finite except for marginal values of $\sigma_0, \sigma'_0, \alpha$, and therefore an arbitrarily large black hole is the generic result of the interconnection of arbitrarily large strings of equal and opposite maximal angular momentum.

The above results can be numerically tested by constructing $\vec{X}_{b,L}$ and $\vec{X}_{b,R}$ with the prescription given in the previous section, and then plotting in σ both $X_b[\sigma, \tau_0] = X_{b,L}[\sigma + \tau_0] + X_{b,R}[\sigma - \tau_0]$ and $Y_b[\sigma, \tau_0] = Y_{b,L}[\sigma + \tau_0] + Y_{b,R}[\sigma - \tau_0]$. One can see that, in two intervals in σ , both X and Y are constant. A sample is shown in figure 4. One can check that for generic values of $\sigma_0, \sigma'_0, \alpha$ the figures are similar.

Finally, let us consider the case in which the interconnecting strings have, in addition to angular momentum, equal and opposite linear momentum along the transverse Z direction. In this case the interconnected strings will in general also stretch in the Z direction (periodically in τ , if one forgets gravity) and therefore the finite fraction of the string described above will not shrink exactly to zero size at τ_0 . In order to conclude that a black hole will still form we have to compare the elongation in Z with the Schwarzschild radius R_s . The maximum elongation in Z is of order Tv , where $T \sim \ell$ is the period of the motion, ℓ is the length of the strings and v is the relative velocity between the centers of mass. Like before, the overall string scale factors out and we get the condition, for a non relativistic

⁵In particular, this means that for strings with masses much larger than the Planck mass (as it is obviously the case for astrophysical cosmic strings) quantum gravity effects can be ignored, since the Schwarzschild radius will be much larger than the Planck scale.

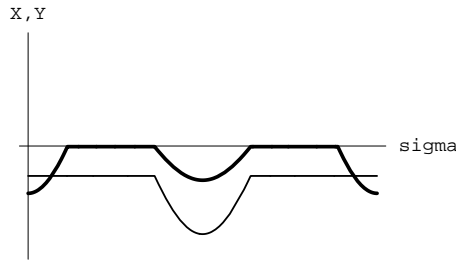


Figure 4: For generic values of the center of mass parameters parameters A, B and relative orientation α , part of the string shrinks becoming a point during the time evolution. In the figure, these are the values of σ for which both X (thicker line) and Y (thinner) are constant.

center-of mass motion of the string, that the ratio of the relative velocity v between the strings to the velocity of light ($c = 1$) should be smaller than $G\mu$ times some number of order 1 depending on σ_0, σ'_0 .

As already mentioned, gravitational collapse of $J_I = -J_{II}$ strings for generic initial data is expected to occur also in the case of the more stable open string which oscillates in the transverse dimension (2.3), provided the string size ℓ_{extra} in the extra dimensions is a small fraction of the overall size L , but still much larger than $l_s = \sqrt{\alpha'}$ to ensure stability. More precisely, $\mu^{-1/2} \ll \ell_{\text{extra}} < G\mu L$ or $\frac{l_s}{L} \ll \theta < G\mu$.

3.2.4 Interconnection of rotating closed strings on $M^4 \times S^1$

Now consider the interconnection of the long-lived closed strings of section 2.1. Let us consider the case of two strings having the same winding in W . For the X, Y coordinates, the solutions of the strings I and II are the same as in eqs. (3.6) and (3.7), now with $\sigma \in [0, 2\pi)$. Similarly, the solutions after interconnection are the same as in eqs. (3.10) and (3.11). The closed strings $\vec{X}_{a,b}(\sigma, \tau) = \vec{X}_{a,bL}(\sigma + \tau) + \vec{X}_{a,bR}(\sigma - \tau)$ are defined for $0 \leq \sigma \leq 2\Delta_{a,b}\sigma$.

For these closed strings, the interconnection takes place at two points σ_0 and $2\pi - \sigma_0$ for the string I and σ'_0 and $2\pi - \sigma'_0$ for the string II.

The intersection in the W coordinate requires that $W_I(\sigma_0) = W_{II}(\sigma'_0)$ and $W_I(2\pi - \sigma_0) = W_{II}(2\pi - \sigma'_0)$, where we take $W_I = R\sigma_I$, $W_{II} = R\sigma_{II}$. This implies $\sigma_0 = \sigma'_0$ and, in turn, $2\Delta_{a,b}\sigma = 2\pi$, consistently with the fact that the strings a and b have both winding numbers equal to 1.

The results of the previous section tell us that for $\tau = \tau_0 = \pi/2 - \alpha/2$ a finite fraction of the arbitrarily large mass of the string undergoes gravitational collapse, for generic values of α .

Classically, the condition $\sigma_0 = \sigma'_0$ implies that rather than a geometric area we get a one-dimensional “cross-section”. Quantum mechanically, however, the interconnection process can also take place if the interconnecting points are separated by a distance of order $\sqrt{\alpha'}$. Therefore the cross section for the collapse of a finite fraction of the string will be of the order of the string length times $\sqrt{\alpha'}$, times g_s^4 .

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A. String splitting and joining: general formalism

A.1 Splitting of closed and open strings

Consider first the splitting of a closed string. The initial closed string is described by the solution

$$X_0 = \alpha' M \tau, \quad \vec{X}_R = \vec{X}_R(\sigma_-), \quad \vec{X}_L = \vec{X}_L(\sigma_+), \quad (\text{A.1})$$

with $\sigma_{\pm} = \sigma \pm \tau$ and $\sigma \in [0, 2\pi)$. In this gauge $X_0 = \alpha' M \tau$, the Virasoro constraints become:

$$(\partial_s \vec{X}_{R,L}(s))^2 = \frac{1}{4} (\alpha' M)^2. \quad (\text{A.2})$$

The momentum of the string is $\alpha' \vec{p} = \vec{p}_R + \vec{p}_L$, with

$$\vec{p}_R = -\frac{\vec{X}_R(2\pi) - \vec{X}_R(0)}{2\pi}, \quad \vec{p}_L = \frac{\vec{X}_R(2\pi) - \vec{X}_R(0)}{2\pi}. \quad (\text{A.3})$$

For a string with winding w around a compact dimension X of radius R , one has $p_R = \frac{1}{2}(\alpha' p - wR)$, $p_L = \frac{1}{2}(\alpha' p + wR)$, so that $p_L - p_R = wR$. If $p_{R,L} \neq 0$ then $X_{R,L}(s)$ is not periodic. In such a case, one can define a periodic function by subtracting the non-periodic part,

$$\begin{aligned} X_R(s) &= \left(X_R(s) + p_R s \right) - p_R s, \\ X_L(s) &= \left(X_L(s) - p_L s \right) + p_L s. \end{aligned} \quad (\text{A.4})$$

The functions within the parentheses (\dots) are continuous – but kinks are allowed – and periodic by definition. This simple observation will be useful for the construction below.

Now we assume that at $\tau = 0$ there is a contact between two points of the closed string and the string breaks into two fragments \vec{X}_I, \vec{X}_{II} ,

$$\vec{X}_I = \vec{X}_{IR}(\sigma_-) + \vec{X}_{IL}(\sigma_+), \quad \vec{X}_{II} = \vec{X}_{IIR}(\sigma_-) + \vec{X}_{IIL}(\sigma_+). \quad (\text{A.5})$$

The fragment solutions are uniquely determined by the condition that the functions X and their first time derivatives are continuous at $\tau = 0$. The first fragment is defined to be the piece of the string with $\sigma_1 < \sigma < \sigma_2$ while the second fragment is the remaining piece $\sigma_2 < \sigma < 2\pi + \sigma_1$. The outgoing strings will carry in general non-zero momentum. Since it is conserved, this can be computed at $\tau = 0$. They are given by $\alpha' \vec{p}_I = \vec{p}_{IR} + \vec{p}_{IL}$,

$$\begin{aligned} \vec{p}_{IR} &= -\frac{\vec{X}_R(\sigma_2) - \vec{X}_R(\sigma_1)}{2\pi}, & \vec{p}_{IL} &= \frac{\vec{X}_L(\sigma_2) - \vec{X}_L(\sigma_1)}{2\pi}, \\ \vec{p}_{IIR} &= -\frac{\vec{X}_R(2\pi + \sigma_1) - \vec{X}_R(\sigma_2)}{2\pi}, & \vec{p}_{IIL} &= \frac{\vec{X}_L(2\pi + \sigma_1) - \vec{X}_L(\sigma_2)}{2\pi}. \end{aligned} \quad (\text{A.6})$$

Consider the general case where there may be compact dimensions of radii R_i , $i = 1, \dots, D - 1$. This includes the uncompact $R = \infty$ case. The breaking is possible if $X_i(\sigma_1, 0) = X_i(\sigma_2, 0) \bmod n_i R_i$ where n_i are integers (there is no summation over i). For uncompact dimensions, $n_i = 0$. This condition can be written as

$$X_{iL}(\sigma_1, 0) - X_{iL}(\sigma_2, 0) = -X_{iR}(\sigma_1, 0) + X_{iR}(\sigma_2, 0) + 2\pi n_i R_i \quad (\text{A.7})$$

or

$$p_{iL}^I = p_{iR}^I + w_i^I R_i \quad (\text{A.8})$$

with $w_i^I = n_i$. The breaking occurs if this condition can be satisfied for all i , for some σ_2 and σ_1 (in some cases, it could be that there are several solutions, i.e. many contact points). Similarly, since $p_{iL} - p_{iR} = w_i R_i$,

$$p_{iL}^{\text{II}} = p_{iR}^{\text{II}} + w_i^{\text{II}} R_i, \quad w_i^I + w_i^{\text{II}} = w_i. \quad (\text{A.9})$$

The energies are

$$E_I = \frac{(\sigma_2 - \sigma_1)}{2\pi} M, \quad E_{\text{II}} = M - E_I = \frac{(2\pi + \sigma_1 - \sigma_2)}{2\pi} M. \quad (\text{A.10})$$

The masses of each of the outgoing fragments are then given by

$$\begin{aligned} M_I^2 &= M^2 \frac{(\sigma_2 - \sigma_1)^2}{4\pi^2} - \vec{p}_I^2, \\ M_{\text{II}}^2 &= M^2 \frac{(2\pi + \sigma_1 - \sigma_2)^2}{4\pi^2} - \vec{p}_{\text{II}}^2. \end{aligned} \quad (\text{A.11})$$

This defines M_I in terms of M_{II} and in terms of the quantum numbers of the original string.

The initial condition uniquely determines the outgoing solutions to be given by

$$\begin{aligned} \vec{X}_{\text{IR}}(s) &= \left[\vec{X}_R(s) - \frac{\vec{X}_R(s_2) - \vec{X}_R(s_1)}{s_2 - s_1} s \right]_{(s_1, s_2)} + \frac{\vec{X}_R(s_2) - \vec{X}_R(s_1)}{s_2 - s_1} s, \\ \vec{X}_{\text{IL}}(s) &= \left[\vec{X}_L(s) - \frac{\vec{X}_L(s_2) - \vec{X}_L(s_1)}{s_2 - s_1} s \right]_{(s_1, s_2)} + \frac{\vec{X}_L(s_2) - \vec{X}_L(s_1)}{s_2 - s_1} s, \end{aligned} \quad (\text{A.12})$$

where we have introduced the symbol $[f(x)]_{(a,b)} \equiv \hat{f}(x)$ as the periodic function defined by $\hat{f}(x + n(b - a)) = \hat{f}(x)$, $x \in [a, b]$ and n is an integer.

Similarly, the second fragment is described by the solution

$$\begin{aligned} \vec{X}_{\text{IIR}}(s) &= \left[\vec{X}_R(s) - \frac{\vec{X}_R(2\pi + s_1) - \vec{X}_R(s_2)}{2\pi + s_1 - s_2} s \right]_{(s_2, 2\pi + s_1)} + \frac{\vec{X}_R(2\pi + s_1) - \vec{X}_R(s_2)}{2\pi + s_1 - s_2} s \\ \vec{X}_{\text{IIL}}(s) &= \left[\vec{X}_L(s) - \frac{\vec{X}_L(2\pi + s_1) - \vec{X}_L(s_2)}{2\pi + s_1 - s_2} s \right]_{(s_2, 2\pi + s_1)} + \frac{\vec{X}_L(2\pi + s_1) - \vec{X}_L(s_2)}{2\pi + s_1 - s_2} s \end{aligned}$$

By the above equations we have required the Left and Right sectors of the string to be the same at $\tau = 0$ as functions of the world-sheet parameter σ in the interval $0 \leq \sigma \leq 2\pi$.

This implies also the continuity of the first derivative in τ at $\tau = 0$ since $\partial_\tau \vec{X}_{\tau=0} = \partial_\sigma \vec{X}_L(\sigma) - \partial_\sigma \vec{X}_R(\sigma)$.

It is convenient to rescale the s variable to have 2π periodic functions. We define, for the fragment I,

$$s = \hat{s} \frac{(s_2 - s_1)}{2\pi} + s_1, \tag{A.13}$$

whereas for the fragment II

$$s = \hat{s} \frac{(2\pi + s_1 - s_2)}{2\pi} + s_2. \tag{A.14}$$

Note that this implies that both σ and τ get rescaled, and that we imposed continuity of the derivative with respect to the unrescaled τ .

The solutions are then as follows:

$$\begin{aligned} \vec{X}_{IR}(s) &= \left[\vec{X}_R(\hat{s} \frac{(s_2 - s_1)}{2\pi} + s_1) + \vec{p}_{IR} \hat{s} \right]_{(0,2\pi)} - \vec{p}_{IR} \hat{s} \\ \vec{X}_{IL}(s) &= \left[\vec{X}_L(\hat{s} \frac{(s_2 - s_1)}{2\pi} + s_1) - \vec{p}_{IL} \hat{s} \right]_{(0,2\pi)} + \vec{p}_{IL} \hat{s} \\ \vec{X}_{IIR}(s) &= \left[\vec{X}_R(\hat{s} \frac{(2\pi + s_1 - s_2)}{2\pi} + s_2) + \vec{p}_{IIR} \hat{s} \right]_{(0,2\pi)} - \vec{p}_{IIR} \hat{s} \\ \vec{X}_{IIL}(s) &= \left[\vec{X}_L(\hat{s} \frac{(2\pi + s_1 - s_2)}{2\pi} + s_1) - \vec{p}_{IIL} \hat{s} \right]_{(0,2\pi)} + \vec{p}_{IIL} \hat{s} \end{aligned}$$

The above construction holds also for open strings (see [22]). In this case one has simply to remember that for an open string $X_R^\mu(\sigma_-) = X_L^\mu(-\sigma_-)$ and that the interval in σ is $[0, \pi)$.

It is useful to explicitly separate the momentum term as follows:

$$X_L^\mu(\sigma_+) = F^\mu(\sigma_+) + \alpha' p^\mu \sigma_+ + c^\mu, \quad X_R^\mu(\sigma_-) = F^\mu(-\sigma_-) - \alpha' p^\mu \sigma_- - c^\mu \tag{A.15}$$

where c^μ is a constant and F^μ is periodic i.e. $F^\mu(s) = F^\mu(s + 2\pi)$ so that

$$X_{\text{open}}^\mu(\sigma, \tau) = F^\mu(\sigma + \tau) + F^\mu(-\sigma + \tau) + 2\alpha' p^\mu \tau. \tag{A.16}$$

A.2 Joining of open strings

Now consider two *open* strings I and II in the CM frame with energies E_I and E_{II} . We assume that at $\tau = 0$ one end of the string I gets in contact with one end of the string II, that is $\vec{X}_I(0, 0) = \vec{X}_{II}(\pi, 0)$. If the two strings join making one final string \vec{X} , that one will have a mass $M = E_I + E_{II}$.

Therefore since $(\partial_{s_I} \vec{X}_{IL,R})^2 = (\alpha' E_I)^2$, $(\partial_{s_{II}} \vec{X}_{IIL,R})^2 = (\alpha' E_{II})^2$ and for the final string $(\partial_s \vec{X}_{L,R})^2 = (\alpha' M)^2$, the worldsheet parameter $0 \leq s \leq 2\pi$ of the final string must be related to the ones of the joining strings by $s_{I,II} = \frac{M}{E_{I,II}} s + c_{I,II}$.

As it has been said in the splitting case, the matching requirement is equivalent to requiring that the resulting $\vec{X}_{L,R}(s)$ is piecewise identical to $\vec{X}_{IL,R}(s_I)$ and $\vec{X}_{IIL,R}(s_{II})$. Therefore we get \vec{X} by the following construction:

$$\vec{X}_{L,R}(s) = \begin{cases} \vec{X}_{IIL,R}(\frac{M}{E_{II}}s) & -\frac{E_{II}}{M}\pi \leq s < \frac{E_{II}}{M}\pi \\ \vec{X}_{IL,R}(\frac{M}{E_I}(s - \frac{E_{II}}{M}\pi)) & \frac{E_{II}}{M}\pi \leq s < 2\pi - \frac{E_{II}}{M}\pi \end{cases} \quad (\text{A.17})$$

Since $\vec{X}_{L,R}(s)$ must be 2π periodic, joining is only possible if

$$\vec{X}_{IIL,R}(-\pi) = \vec{X}_{IL,R}((\frac{M}{E_I} - \frac{E_{II}}{E_I})2\pi) .$$

As a particular case, consider two strings of equal mass, carrying equal and opposite momenta, described by the solutions

$$\begin{aligned} X_{0I} = 2\alpha' E\tau, \quad \vec{X}_{IL}(s) = \vec{F}_I(s) + \alpha'\vec{p}\left(s - \frac{\pi}{2}\right), \quad \vec{X}_{IR}(s) = \vec{F}_I(-s) - \alpha'\vec{p}\left(s - \frac{\pi}{2}\right) \\ X_{0II} = 2\alpha' E\tau \quad \vec{X}_{IIL}(s) = \vec{F}_{II}(s) - \alpha'\vec{p}\left(s - \frac{\pi}{2}\right), \quad \vec{X}_{IIR}(s) = \vec{F}_{II}(-s) + \alpha'\vec{p}\left(s - \frac{\pi}{2}\right) \end{aligned} \quad (\text{A.18})$$

where $\vec{F}_{I,II}(s)$ is 2π -periodic and by assumption $(\partial_s \vec{F}_I + \alpha'\vec{p})^2 = (\partial_s \vec{F}_{II} - \alpha'\vec{p})^2 = (\alpha'E)^2$. The joining condition $\vec{X}_I(0,0) = \vec{X}_{II}(\pi,0)$ implies $\vec{F}_I(0) = \vec{F}_{II}(\pi)$.

Explicitly, in this case $E_I = E_{II} = M/2$, the resulting solution after the joining is given by

$$\vec{X}_{L,R}(s) = \begin{cases} \vec{X}_{IIL,R}(2s) & -\frac{\pi}{2} \leq s < \frac{\pi}{2} \\ \vec{X}_{IL,R}(2s - \pi) & \frac{\pi}{2} \leq s < \frac{3\pi}{2} \end{cases} \quad (\text{A.19})$$

Note that $\vec{X}_{L,R}(-\frac{\pi}{2}) = \vec{X}_{L,R}(\frac{3\pi}{2})$ and that $X^\mu(\sigma,\tau)$ has the open string structure (A.16) with zero momentum. Outside the interval $-\frac{\pi}{2} \leq s < \frac{3\pi}{2}$, $\vec{X}_{L,R}(s)$ is defined by its periodic extension, i.e. by replacing s by $\hat{s} = s - [s/2\pi]$.

The resulting string being periodic, after one period $\Delta\tau = 2\pi$ it comes back to the original configuration. Being an open string, it could split again at anytime. For example, at $\Delta\tau = 2\pi$ it could split into the two original pieces $\vec{X}_{I,II}$ or else continue in its periodic motion.

As an application, consider now the case where two open strings with maximum angular momentum move in the same clockwise sense. The solutions are $X(\tau,\sigma) = X_L(\tau + \sigma) + X_L(\tau - \sigma)$ with

$$X_{IL}(s) = \frac{L}{2} \cos s, \quad Y_{IL} = \frac{L}{2} \sin s, \quad (\text{A.20})$$

$$X_{IIL}(s) = L + \frac{L}{2} \cos s, \quad Y_{IL} = \frac{L}{2} \sin s. \quad (\text{A.21})$$

The main difference with respect to case of opposite angular momenta discussed in section 3.1 is that, at the moment of the joining at $\tau = 0$, the ends are now moving with opposite velocities. In the previous case, they were moving with the same velocity and, as a result, the string which resulted after the joining was smooth. Now, because the attached ends are moving at opposite velocities, a kink will be formed. This case illustrates that the

formation of kinks in string joining (as in string splitting [22]) is generic, since the generic situation is that the velocities of the two joined ends are different (as vectors, the endpoints of open strings always move at the speed of light).

The solution after the joining can be constructed using eq. (A.19). Beside the formation of the kink, another interesting feature is that the open strings become folded cyclically during the evolution.

A.3 Joining of closed strings

Consider two closed strings I and II described by the solutions

$$\begin{aligned} X_{0I} &= \alpha' E_I \tau, & \vec{X}_{IR} &= \vec{X}_{IR}(\sigma_-), & \vec{X}_{IL} &= \vec{X}_{IL}(\sigma_+), \\ X_{0II} &= \alpha' E_{II} \tau, & \vec{X}_{IIR} &= \vec{X}_{IIR}(\sigma_-), & \vec{X}_{IIL} &= \vec{X}_{IIL}(\sigma_+). \end{aligned} \quad (\text{A.22})$$

We assume that at $\tau = 0$ one point of the closed string I (say $\sigma_I = 0$) gets in contact with one point of the closed string II (say $\sigma_{II} = \pi$), and the strings join. The resulting solution is again uniquely determined by the assumption of continuity of X and \dot{X} at $\tau = 0$.

Now the solution after the joining is

$$X^\mu(\sigma, \tau) = X_L^\mu(\sigma + \tau) + X_R^\mu(-\sigma + \tau), \quad (\text{A.23})$$

where $X_L^\mu(\sigma + \tau)$ and $X_R^\mu(-\sigma + \tau)$ are determined by continuity of X and \dot{X} at $\tau = 0$. When the energies are the same, we find that $X_L^\mu(\sigma + \tau)$ and $X_R^\mu(-\sigma + \tau)$ are given by eq. (A.19). When they are different, the solution is constructed in a similar way as (A.17).

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